Thermal diffusion by stochastic electromagnetic fluctuations

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Background

1. In experiments, **electrostatic and magnetic fluctuations usually coexist. Under their simultaneous influences electrons and ions will diffuse.**

electrostatic fluctuations	:	ITG
magnetic fluctuations	:	equilibrium,
		instabilities

2. In simulations, **gyrokinetic self-consistent electromagnetic turbulence simulations in systems with electrons and ions are quite difficult (at least 5 years later)**

Purposes

To develop a new simple method to evaluate the thermal diffusion by coexisting stochastic electrostatic and magnetic fluctuations, where

1. **both electrostatic and magnetic fluctuations are treated in the same framework,**
2. **analytical expression of the diffusion coefficient is obtained,**
3. **intuitive interpretation of the diffusion in experiments and simulations is given.**

Object

turbulent level $\Rightarrow$ depends on configuration
diffusion level $\Rightarrow$ depends on turbulent level

1. **geometry:**
   A large aspect straight helical or tokamak system with low-β and small gyro-radii
   only passing particles considered (Trapped particles can be treated in terms of action-angle variables)
2. **fluctuations:**
   given coexisting homogeneous stochastic electrostatic and magnetic fluctuations
   statistical properties of fluctuations:
   Gaussian with a finite correlation time and no mean value

Contents

1. derivation of mono-energetic diffusion coefficient
   Drift Kinetic Deterministic Equations is regarded as **Stochastic Differential Equations (SDE) by stochastic instability of orbits**
   renomalization of Lagrangian auto-correlation function by using perturbed orbits
   realization of the stochastic instability by replacing discrete parallel wave number by continuous parallel wave number (based on J.A.Krommes)
2. thermal diffusion coefficients of electrons and ions (velocity space integration)
3. future works

1 Derivation of mono-energetic diffusion coefficient

1.1 **Drift Kinetic Deterministic Equations** is regarded as **Stochastic Differential Equations (SDE) by stochastic instability of orbits**

Drift Kinetic Deterministic Equations (by Little John)

$$\vec{v} \;=\; v_{||}\frac{\vec{B}+\delta\vec{B}+\nabla\times(\rho_{||}\vec{B})}{B+\delta B_{||}+\rho_{||}J_{||}},\;\; \hat{n} \;\equiv\; \frac{\vec{B}}{B},\;\; \rho_{||} \;\equiv\; \frac{v_{||}}{\Omega},\;\; \Omega \;\equiv\; \frac{eB}{m},\;\; \delta\vec{B} \;=\; \nabla\times\delta\vec{A},\;\; \delta\vec{A}=\alpha\vec{B}=\delta A_{||}\hat{n}.$$

In a large aspect helical or tokamak system with low-β, small gyro-radii, and a model field $B = B[1-\varepsilon_t\cos\theta-\varepsilon_h\cos(L\theta-M\zeta)]$

assuming $\dfrac{\delta B_r}{B} \gg \dfrac{\rho}{R}$ **and** $\dfrac{\delta E_\theta}{vB} \gg \dfrac{\rho}{R}$ **(toroidal drifts by ∇B and $\vec{\kappa}$ are neglected)**

$$\frac{dr}{dt} \sim \underbrace{v_{||}\frac{1}{rB}\frac{\partial\delta A_{||}}{\partial\theta}-\frac{1}{rB}\frac{\partial\delta\phi}{\partial\theta}}_{\textbf{part due to fluctuations}},$$

$$\frac{d\theta}{dt} \sim \underbrace{v_{||}\frac{\iota\!\!-}{R}-\omega_{E\times B}}_{\textbf{part without fluctuations}} - \underbrace{v_{||}\frac{1}{rB}\frac{\partial\delta A_{||}}{\partial r}+\frac{1}{rB}\frac{\partial\delta\phi}{\partial r}}_{\textbf{part due to fluctuations}},$$

$$\frac{d\zeta}{dt} \sim v_{||}\frac{1}{R}.$$

parts due to fluctuations $\Rightarrow$ **stochastic parts**

Slow particles mainly contribute to diffusion by fluctuations, fast ones mainly does to neoclassical diffusion, if collisions exist.(partition of velocity space)

Stochastic Differential Equations (SDE) by the stochastic instability of orbits

$$
\begin{aligned}
\frac{dr}{dt} &\sim \underbrace{\tilde{g}_r(\vec{r}(t),t)}_{\textbf{stochastic part}}, \\
\frac{d\theta}{dt} &\sim \underbrace{v_{||}\frac{\iota}{R} - \omega_{E\times B}}_{\textbf{deterministic part}} - \underbrace{\tilde{g}_\theta(\vec{r}(t),t)}_{\textbf{stochastic part}}, \\
\frac{d\zeta}{dt} &\sim v_{||}\frac{1}{R},
\end{aligned}
$$

where **statistical properties of $\tilde{g}_r(\vec{r}(t),t)$ and $\tilde{g}_\theta(\vec{r}(t),t)$ are assumed to be Gaussian with no mean value**

$$
\begin{aligned}
\tilde{g}_r(\vec{r}(t),t) &\equiv v_{||}\frac{1}{rB}\frac{\partial \delta A_{||}}{\partial \theta} - \frac{1}{rB}\frac{\partial \delta\phi}{\partial\theta}, \quad \delta A_{||} = \textstyle\sum_{mn} \delta A_{||mn}(r)\cos\left[n\zeta - m\theta + \delta_{mn}^{(\delta A)} - \omega_{mn}^{(\delta A)} t\right], \\
\tilde{g}_\theta(\vec{r}(t),t) &\equiv v_{||}\frac{1}{rB}\frac{\partial \delta A_{||}}{\partial r} - \frac{1}{rB}\frac{\partial \delta\phi}{\partial r}, \quad \delta\phi = \textstyle\sum_{mn} \delta\phi_{mn}(r)\cos\left[n\zeta - m\theta + \delta_{mn}^{(\delta\phi)} - \omega_{mn}^{(\delta\phi)} t\right].
\end{aligned}
$$

simplification **by locality of the radial diffusion**

$$
\begin{aligned}
\frac{dr}{dt} &\sim \underbrace{\tilde{g}_r(r=r(t_0),\theta(t),\zeta(t),t)}_{\textbf{stochastic part}}, \\
\frac{d\theta}{dt} &\sim \underbrace{\left[v_{||}\frac{\iota}{R} - \omega_{E\times B}\right]_{r=r(t_0)}}_{\textbf{deterministic part}} - \underbrace{\tilde{g}_\theta(r=r(t_0),\theta(t),\zeta(t),t)}_{\textbf{stochastic part}}, \\
\frac{d\zeta}{dt} &\sim v_{||}\frac{1}{R},
\end{aligned}
$$

1.2 The formal solution of SDE

$$r(t) = r(t_0) + \underbrace{\int_{t_0}^{t} d\tau \tilde{g}_r(r = r(t_0), \theta(\tau), \zeta(\tau), \tau),}_{\textbf{perturbed orbits}}$$

$$\theta(t) = \theta(t_0) + \left[v_{||}\frac{\iota}{R} - \omega_{E\times B}\right]_{r(t)=r(t_0)} (t - t_0) + \underbrace{\int_{t_0}^{t} d\tau \tilde{g}_\theta(r = r(t_0), \theta(\tau), \zeta(\tau), \tau),}_{\textbf{perturbed orbits}}$$

$$\zeta(t) = \zeta(t_0) + v_{||}\frac{1}{R}(t - t_0).$$

1.3 Renormalized mono-energetic diffusion coefficient at $r = r_0$

$$D_\theta(t, t_0) = \frac{1}{2}\frac{d}{dt}\left\langle (\theta(t) - \langle\theta(t)\rangle)^2 \right\rangle = \frac{1}{2}\frac{d}{dt}\left\langle \left[\int_{t_0}^{t} d\tau \left[\frac{d\theta(\tau)}{d\tau} - \left\langle \frac{d\theta(\tau)}{d\tau}\right\rangle\right]\right]^2 \right\rangle$$

$$= \int_{t_0}^{t} d\tau \left\langle \tilde{g}_\theta(r = r(t_0), \theta(t), \zeta(t), t)\tilde{g}_\theta(r = r(t_0), \theta(\tau), \zeta(\tau), \tau)\right\rangle = \int_{t_0}^{t} d\tau \mathcal{R}_\theta(t, \tau) = \mathcal{F}_\theta\left[D_\theta(t, t_0)\right]$$

unperturbed orbits of $\theta(t)$ in $\tilde{g}_\theta$ $\Rightarrow$ quasi-linear diffusion of $D_\theta(t, t_0)$

perturbed orbits of $\theta(t)$ in $\tilde{g}_\theta$ $\Rightarrow$ renormalized diffusion of $D_\theta(t, t_0)$

$\mathcal{R}_\theta(t, \tau)$ **: Lagrangian auto-correlation function of the velocity**

After obtaining $D_\theta(t, t_0)$, $D_r(t, t_0)$ is obtained.

1.4 Treatment of stochastic parts

By substituting the solution of SDE into $\mathcal{R}_\theta(t,\tau)$, an ensemble average appears:

$$\left\langle e^{\pm i\xi}\right\rangle, \qquad \xi \equiv a\int_{t_0}^{t} dt_1\tilde{g}_\theta(r=r(t_0),\theta(t_1),\zeta(t_1),t_1) + b\int_{t_0}^{\tau} dt_2\tilde{g}_\theta(r=r(t_0),\theta(t_2),\zeta(t_2),t_2)$$

By using cumulant C_l expansion and **Gaussianity with no mean value : $C_{l=1}=C_{l\geq 3}=0$:**

$$\left\langle e^{\pm i\xi}\right\rangle = \exp\left\{\sum_{l=1}^{\infty}\frac{(\pm i)^l}{l}C_l\right\} = e^{-\frac{1}{2}\left\langle \xi^2\right\rangle}$$

$$\left\langle \xi^2\right\rangle = a^2\int_{t_0}^{t}dt_1\int_{t_0}^{t}dt_1'\mathcal{R}_\theta(t_1,t_1') + b^2\int_{t_0}^{\tau}dt_1\int_{t_0}^{\tau}dt_1'\mathcal{R}_\theta(t_1,t_1') + 2ab\int_{t_0}^{t}dt_1\int_{t_0}^{\tau}dt_1'\mathcal{R}_\theta(t_1,t_1'),$$

Long term limit, $\mathcal{R}_\theta(t,\tau)$ becomes stationary with a finite correlation time τ_{ac}^θ:

$$\mathcal{R}_\theta(t,\tau) \sim \mathcal{R}_\theta(t-\tau) \sim \frac{D_\theta}{\tau_{ac}^\theta}\exp\{-\frac{t-\tau}{\tau_{ac}^\theta}\}, \quad \tau_{ac}^\theta \sim \frac{1}{\overline{m}^2 D_\theta},$$

$$\text{For } \; t-\tau \gg \tau_{ac}^\theta, \qquad \frac{1}{2}\left\langle \xi^2\right\rangle = a^2(t-t_0)+(2ab+b^2)(\tau-t_0)$$

$$\left\langle e^{\pm i\xi}\right\rangle = \exp\left\{-\left[a^2(t-t_0)+(2ab+b^2)(\tau-t_0)\right]\right\}.$$

By time integration and picking up the non-damping terms,

1. **only Fourier modes with $m'=m$ and $n'=n$ remain in $\sum_{mn}\sum_{m'n'}$,**
2. **the dependence on the initial conditions vanishes.**

1.5 Renormalized mono-energetic diffusion coefficient in the long term limit

$$D_\theta = \lim_{t-t_0 \gg \tau_{ac}^\theta} \int_{t_0}^{t} d\tau \mathcal{R}_\theta(t-\tau)$$

$$\sim \frac{1}{2}\sum_{mn}\left[v_{||}\frac{1}{rB}\frac{\partial \delta A_{||mn}}{\partial r}\right]^2 \frac{m^2 D_\theta}{\left[k_{||}v_{||} + m\omega_{E\times B} - \omega_{mn}^{(\delta A)}\right]^2 + \underbrace{\left[m^2 D_\theta\right]^2}_{\textbf{renormalized part}}}$$

$$+ \frac{1}{2}\sum_{mn}\left[\frac{1}{rB}\frac{\partial \delta\phi_{mn}}{\partial r}\right]^2 \frac{m^2 D_\theta}{\left[k_{||}v_{||} + m\omega_{E\times B} - \omega_{mn}^{(\delta\phi)}\right]^2 + \underbrace{\left[m^2 D_\theta\right]^2}_{\textbf{renormalized part}}}, \quad \text{where} \quad k_{||} = \frac{n - m\not{\iota}}{R}.$$

$$D_r \sim \left(\frac{r\overline{k}_\theta}{\overline{k}_r}\right)^2 D_\theta, \qquad \overline{k}_r \sim \frac{1}{\delta A_{||mn}}\frac{\partial \delta A_{||mn}}{\partial r} \sim \frac{1}{\delta\phi_{||mn}}\frac{\partial \delta\phi_{||mn}}{\partial r}, \qquad r\overline{k}_\theta \sim \overline{m}.$$

with typical value of Q expressed by $\overline{Q}$

1. **The velocity dependence is different between magnetic and electrostatic fluctuations (coming from the equation of motion).**
2. **Connection between D_r and D_θ is difficult when $\overline{k}_r^{(\delta A)}$ is completely different from $\overline{k}_r^{(\delta\phi)}$.** Hereafter, **the spectrum of $\delta A_{||}$ is assumed to be fairly similar to $\delta\phi$.**

1.6 Realization of the stochastic instability

1. **Stochastic instability of orbits is brought by influences of simultaneous multiple waves on orbits. Particles feel infinite number of waves along perturbed orbits.**

2. To express this situation, **the finite summation of the discrete** parallel wave number is replaced by the **integration of the continuous** parallel wave number (based on J.A.Krommes)

$$\sum_{mn} = \sum_{mk_{||}} \Rightarrow \sum_{m} \frac{1}{\Delta k_{||}} \int_{\delta k_{||min}(<0)}^{\delta k_{||max}(>0)} dk_{||}$$

$\delta k_{||max}, \delta k_{||min}$ **: the maximum and minimum parallel wave numbers contributing to the diffusion around** $(k_{||} = 0)$、except for boundaries of the stochastic region,

$$-\delta k_{||min} \sim \delta k_{||max} = \delta k_{||} = \left| \frac{k_\theta}{k_r L_s} \right|, \quad L_s = \frac{R}{\iota\!\!\!-|s|}, \quad s = \frac{r}{q}\frac{dq}{dr},$$

On $\Delta k_{||}$ $$\langle Q \rangle = \frac{1}{N} \sum_{i=1}^{N} Q_i \sim \frac{\displaystyle\int QW dk_{||}}{\displaystyle\int W dk_{||}} \Rightarrow \sum_{i=1}^{N} Q_i \sim \frac{\displaystyle\int QW dk_{||}}{\dfrac{\displaystyle\int W dk_{||}}{N}}, \quad W : \text{the envelop of } k_{||}$$

$$\Delta k_{||} = \frac{\displaystyle\int_{\delta k_{||min}}^{\delta k_{||max}} W dk_{||}}{N} \sim \frac{1}{qR}, \quad L_{||} \equiv \frac{2\pi}{\Delta k_{||}} \sim 2\pi q R \ :$$ **parallel correlation length**.

The resultant mono-energetic diffusion coefficient:

$$D_\theta = \frac{L_{||}}{4\pi}\sum_m \int_{-\delta k_{||}}^{\delta k_{||}} dk_{||}\left[v_{||}\frac{1}{rB}\frac{\partial \delta A_{||mk_{||}}}{\partial r}\right]^2 \frac{m^2 D_\theta}{\left[k_{||}v_{||}+m\omega_{E\times B}-\omega_{mk_{||}}^{(\delta A)}\right]^2+[m^2 D_\theta]^2}$$
$$+ \frac{L_{||}}{4\pi}\sum_m \int_{-\delta k_{||}}^{\delta k_{||}} dk_{||}\left[\frac{1}{rB}\frac{\partial \delta \phi_{mk_{||}}}{\partial r}\right]^2 \frac{m^2 D_\theta}{\left[k_{||}v_{||}+m\omega_{E\times B}-\omega_{mk_{||}}^{(\delta \phi)}\right]^2+[m^2 D_\theta]^2}.$$

By assuming moderate variations of the amplitude and the frequency:

$$D_\theta \sim \frac{L_{||}}{4\pi}\sum_m \left\langle\left[v_{||}\frac{1}{rB}\frac{\partial \delta A_{||mk_{||}}}{\partial r}\right]^2\right\rangle_{k_{||}} \int_{-\delta k_{||}}^{\delta k_{||}} dk_{||}\frac{m^2 D_\theta}{\left[k_{||}v_{||}+m\omega_{E\times B}-\left\langle\omega_{mk_{||}}^{(\delta A)}\right\rangle_{k_{||}}\right]^2+[m^2 D_\theta]^2}$$
$$+ \frac{L_{||}}{4\pi}\sum_m \left\langle\left[\frac{1}{rB}\frac{\partial \delta \phi_{mk_{||}}}{\partial r}\right]^2\right\rangle_{k_{||}} \int_{-\delta k_{||}}^{\delta k_{||}} dk_{||}\frac{m^2 D_\theta}{\left[k_{||}v_{||}+m\omega_{E\times B}-\left\langle\omega_{mk_{||}}^{(\delta \phi)}\right\rangle_{k_{||}}\right]^2+[m^2 D_\theta]^2}$$

where $\langle Q\rangle_{k_{||}}$ means the replacement of the $k_{||}$ dependence by the typical values at the initial position. Remaining integrations with respect to $k_{||}$ are analytically integrated.

The final form of mono-energetic diffusion coefficient:

$$D_\theta(v_{||}, \hat{\omega}_m^{(\delta A)}, \hat{\omega}_m^{(\delta\phi)}) \sim \frac{L_{||}}{4\pi} \sum_m \sum_{\pm} \left\langle v_{||} \left[\frac{1}{rB} \frac{\partial \delta A_{||mk_{||}}}{\partial r} \right]^2 \right\rangle_{k_{||}} \mathrm{Tan}^{-1} \left[\frac{\delta k_{||} v_{||} \pm \hat{\omega}_m^{(\delta A)}}{m^2 D_\theta} \right]$$
$$+ \frac{L_{||}}{4\pi} \sum_m \sum_{\pm} \left\langle \frac{1}{v_{||}} \left[\frac{1}{rB} \frac{\partial \delta \phi_{mk_{||}}}{\partial r} \right]^2 \right\rangle_{k_{||}} \mathrm{Tan}^{-1} \left[\frac{\delta k_{||} v_{||} \pm \hat{\omega}_m^{(\delta\phi)}}{m^2 D_\theta} \right]$$

where $\hat{\omega}_m^{(\delta A)} \equiv \left\langle \omega_{mk_{||}}^{(\delta A)} \right\rangle_{k_{||}} - m\omega_{E\times B}, \qquad \hat{\omega}_m^{(\delta\phi)} \equiv \left\langle \omega_{mk_{||}}^{(\delta\phi)} \right\rangle_{k_{||}} - m\omega_{E\times B},$

$$D_\theta(-v_{||}, \hat{\omega}_m^{(\delta A)}, \hat{\omega}_m^{(\delta\phi)}) = D_\theta(v_{||}, \hat{\omega}_m^{(\delta A)}, \hat{\omega}_m^{(\delta\phi)}),$$
$$D_\theta(v_{||}, -\hat{\omega}_m^{(\delta A)}, -\hat{\omega}_m^{(\delta\phi)}) = D_\theta(v_{||}, \hat{\omega}_m^{(\delta A)}, \hat{\omega}_m^{(\delta\phi)}).$$

In order to obtain limiting cases, $\mathrm{Tan}^{-1}x$ is approximated by

$$\mathrm{Tan}^{-1}x \sim \begin{cases} \frac{\pi}{2} & \text{for } x \geq \frac{\pi}{2} \\ x & \text{for } |x| \leq \frac{\pi}{2} \\ -\frac{\pi}{2} & \text{for } x \leq -\frac{\pi}{2} \end{cases}$$

By using above approximation, the velocity space integration will be done.

2 Thermal diffusion coefficient

2.1 Cases with only magnetic fluctuations in low frequency limit $\hat{\omega}_m^{(\delta A)} \sim 0$

Scale separator for magnetic fluctuations : $\mathcal{R}_M$

$$\mathcal{R}_M \equiv \left[\frac{\pi}{8} \frac{L_{||}\overline{k}_r^2}{\delta k_{||}} \sum_m \left\langle \left(\frac{\delta B_{rmk_{||}}}{B} \right)^2 \right\rangle_{k_{||}} \right]^{1/2}$$

$$\mathcal{R}_M \sim \frac{\textbf{displacements by diffusion}}{\textbf{correlation length of fluctuations}}, \text{ for } \delta k_{||} \sim L_{||}^{-1}, \overline{k}_r \sim L_{\perp}^{-1}$$

$L_{\perp}$: **perpendicular correlation length of the fluctuations**

$\mathcal{R}_M \ll 1$ **scale separable: quasi-linear limit**

1. **low amplitude limit (quasi-linear limit), $\mathcal{R}_M \leq 1$**

$$D_r^{(\alpha)}(v_{||}) \sim \frac{L_{||}}{4} |v_{||}| \sum_m \left\langle \left(\frac{\delta B_{rmk_{||}}}{B} \right)^2 \right\rangle_{k_{||}}$$

Averaged (unperturbed) orbits are good approximation.

2. **high amplitude limit, $\mathcal{R}_M \geq 1$**

$$D_r^{(\alpha)}(v_{||}) \sim |v_{||}| \sqrt{\frac{L_{||}}{2\pi} \sum_m \left\langle \left(\frac{\delta B_{rmk_{||}}}{B} \right)^2 \right\rangle_{k_{||}} \frac{\delta k_{||}}{\overline{k}_r^2}}.$$

Diffusive (perturbed) orbits are good approximation.

$$D_r^{(\alpha)}(v_{||}) \sim \frac{v_{||}}{v_{T\alpha}} \frac{2 v_{T\alpha} \delta k_{||}}{\pi \overline{k}_r^2} \begin{cases} \mathcal{R}_M^2 & \text{for } \mathcal{R}_M \le 1 \\ \mathcal{R}_M & \text{for } \mathcal{R}_M \ge 1 \end{cases}$$

$$\chi_M^{(\alpha)} \sim \frac{4 v_{T\alpha} \delta k_{||}}{\pi^{3/2} \overline{k}_r^2} \begin{cases} \mathcal{R}_M^2 & \text{for } \mathcal{R}_M \le 1 \\ \mathcal{R}_M & \text{for } \mathcal{R}_M \ge 1 \end{cases}, \quad \chi_M^{(e)} \sim \left(\frac{m_i}{m_e}\right)^{1/2} \chi_M^{(i)}$$

2.2 Cases with only electrostatic fluctuations in low frequency limit $\hat{\omega}_m^{(\delta\phi)} \sim 0$

Scale separator for electrostatic fluctuations : $\mathcal{R}_E^{(\alpha)}$

$$\mathcal{R}_E^{(\alpha)} \equiv \left[\frac{\pi}{8} \frac{L_{||} \overline{k}_r^2}{\delta k_{||}} \sum_m \left\langle \left(\frac{\delta E_{\theta m k_{||}}}{B v_{T\alpha}} \right)^2 \right\rangle_{k_{||}} \right]^{1/2}$$

$$\mathcal{R}_E^{(\alpha)} \sim \frac{\textbf{displacements by diffusion}}{\textbf{correlation length of fluctuations}}, \quad \text{for } \delta k_{||} \sim L_{||}^{-1}, \overline{k}_r \sim L_{\perp}^{-1}$$

$L_\perp$: **perpendicular correlation length of the fluctuations**

$\mathcal{R}_E^{(\alpha)} \ll 1$ **scale separable: quasi-linear limit**

1. **low amplitude and/or high velocity limit,** $\mathcal{R}_E^{(\alpha)} \le \dfrac{v_{||}}{v_{T\alpha}}$

$$D_r^{(\alpha)}(v_{||}) \sim \frac{L_{||}}{4|v_{||}|} \sum_m \left\langle \left(\frac{\delta E_{\theta m k_{||}}}{B} \right)^2 \right\rangle_{k_{||}}$$

Averaged (unperturbed) orbits are good approximation.

2. **high amplitude and/or low velocity limit, $\mathcal{R}_E^{(\alpha)} \geq \frac{v_{||}}{v_{T\alpha}}$**

$$D_r^{(\alpha)}(v_{||}) \sim \sqrt{\frac{L_{||}}{2\pi}\sum_m \left\langle \left(\frac{\delta E_{\theta m k_{||}}}{B}\right)^2 \right\rangle_{k_{||}} \frac{\delta k_{||}}{\overline{k}_r^2}}$$

Diffusive (perturbed) orbits are good approximation.
Due to the velocity dependence, only quasi-linear treatment is not enough to obtain the diffusion coefficient. cf. diffusion by magnetic fluctuations

$$D_r^{(\alpha)}(v_{||}) \sim \frac{2v_{T\alpha}\delta k_{||}}{\pi \overline{k}_r^2} \begin{cases} \left(\mathcal{R}_E^{(\alpha)}\right)^2 \frac{v_{T\alpha}}{v_{||}} & \text{for } \mathcal{R}_E^{(\alpha)} \leq \frac{v_{||}}{v_{T\alpha}} \\ \mathcal{R}_E^{(\alpha)} & \text{for } \mathcal{R}_E^{(\alpha)} \geq \frac{v_{||}}{v_{T\alpha}} \end{cases}$$

$$\chi_E^{(\alpha)} \sim \frac{4v_{T\alpha}\delta k_{||}}{\pi^{3/2}\overline{k}_r^2} \left\{ \mathcal{R}_E^{(\alpha)} \int_0^{\mathcal{R}_E^{(\alpha)}} dx(1+x^2)e^{-x^2} + \left(\mathcal{R}_E^{(\alpha)}\right)^2 \int_{\mathcal{R}_E^{(\alpha)}}^{\infty} dx \left(\frac{1}{x}+x\right) e^{-x^2} \right\},$$

$$\mathcal{R}_E^{(e)} \sim \left(\frac{m_i}{m_e}\right)^{1/2} \mathcal{R}_E^{(i)} \ll \mathcal{R}_E^{(i)}$$

Due to difference of the velocity dependence, $\chi_E^{(\alpha)}$ has more complex nonlinear dependence on fluctuating amplitudes than $\chi_M^{(\alpha)}$.

2.3 Cases with coexisting electrostatic and magnetic fluctuations in low frequency limit $\hat{\omega}_m^{(\delta A)}, \hat{\omega}_m^{(\delta\phi)} \sim 0$

$$\left(\frac{\mathcal{R}_E^{(\alpha)}}{\mathcal{R}_M}\right)^2 \sim \frac{\sum_m \left\langle \left(\frac{\delta E_{\theta m k_{||}}}{v_{T\alpha} B}\right)^2 \right\rangle_{k_{||}}}{\sum_m \left\langle \left(\frac{\delta B_{\theta r k_{||}}}{B}\right)^2 \right\rangle_{k_{||}}} \sim \left(\frac{c}{v_{T\alpha}}\right)^2 \frac{\frac{\varepsilon_0}{2}\sum_m \left\langle \left(\frac{\delta E_{\theta m k_{||}}}{B}\right)^2 \right\rangle_{k_{||}}}{\frac{1}{2\mu_0}\sum_m \left\langle \left(\frac{\delta B_{\theta r k_{||}}}{B}\right)^2 \right\rangle_{k_{||}}} = \left(\frac{c}{v_{T\alpha}}\right)^2 \delta$$

$$\delta \equiv \frac{\frac{\varepsilon_0}{2}\sum_m \left\langle \left(\frac{\delta E_{\theta m k_{||}}}{B}\right)^2 \right\rangle_{k_{||}}}{\frac{1}{2\mu_0}\sum_m \left\langle \left(\frac{\delta B_{\theta r k_{||}}}{B}\right)^2 \right\rangle_{k_{||}}}$$

1. $\delta \sim 1, \quad \mathcal{R}_E^{(i)} \gg \mathcal{R}_E^{(e)} \gg \mathcal{R}_M$
   Both diffusions of electrons and ions are due to electrostatic fluctuations
2. $\delta \sim \frac{m_e}{m_i}, \quad \mathcal{R}_E^{(i)} \gg \mathcal{R}_M \gg \mathcal{R}_E^{(e)}$
   Diffusion of electrons (ions) is due to magnetic (electrostatic) fluctuations
3. $\delta \ll \frac{m_e}{m_i}$
   Both diffusions of electrons and ions are due to magnetic fluctuations

3 Summary and future works

Under the most basic situations that the transport of electrons and ions by the coexisting stochastic electrostatic and magnetic fluctuations are regarded as the normal diffusion,

1. **such a simple synergetic method is developed that the diffusion processes by coexisting stochastic electrostatic and magnetic fluctuations are obtained in the same framework**.
2. **an analytical expression of the diffusion coefficient is obtained in terms of the scale separator.**
3. **properties of the thermal diffusion under coexisting electrostatic and magnetic fluctuations are clarified.**
4. **in the experimental situations, χ_e is governed by magnetic fluctuations and χ_i is governed by electrostatic fluctuations**

remaining problems

1. **ambipolar conditions**
2. **effects of the toroidicity (partition of the velocity space integration)**
3. **effects of the magnetic shear ι' and $\omega_{E\times B}$ shear**